Theory of quantum dot spin lasers

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We formulate a model of a semiconductor quantum dot laser with injection of spin-polarized electrons. As compared to higher-dimensionality structures, the quantum dot based active region is known to improve laser properties, including the spin-related ones. The wetting layer, from which carriers are captured into the active region, acts as an intermediate level that strongly influences the lasing operation. The finite capture rate leads to an increase in lasing thresholds and to saturation of emitted light at higher injection. In spite of these issues, the advantageous threshold reduction, resulting from spin injection, can be preserved. The "spin-filtering" effect, i.e., circularly polarized emission at even modest spin polarization of injection, remains present as well. Our rate-equations description allows to obtain analytical results and provides transparent guidance for improvement of spin lasers.

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I. INTRODUCTION

Experiments on semiconductor spin lasers have demonstrated the potential of spintronics to go beyond the limits of devices relying solely on the carrier charge.[1–4] These structures offer a practical path to realize spintronic devices, which could be useful for communications and signal processing, rather than limited to magnetoresistive effects. Spin injection into lasers is implemented optically, when circularly polarized light imparts the photons' angular momentum to the spin of carriers,[5,6] or electrically, when a magnetic contact polarizes carriers entering the semiconductor.[1] Apart from the successful early demonstration of a spin laser based on a bulklike layer of GaAs,[7] most experiments in this field concentrated on structures with quantum-well (QW) active regions, using optical pump,[2,8–10] electrical injection,[3] or a combination of both.[4,11] Recently, however, an (In,Ga)As/GaAs quantum dot (QD) spin laser with electrical injection has been demonstrated, lasing at temperatures 100 K higher than its QW counterparts.[12] QDs close a succession of reduced-dimensionality structures: quantum wells and wires, which have replaced bulklike active regions of semiconductor lasers.[13] They allow to control the number and spin of carriers, as well as the quantum-confinement geometry.[14] A quantum dot spin laser combines the potential of spin-polarized injection with the advantages of a QD-based active region, such as low threshold, robust temperature performance, and narrow gain spectra.[15,16] In addition to these properties of conventional (spin-unpolarized) QD lasers, the long spin-relaxation times,[17] characteristic for QDs, are advantageous for spin lasers.

Spin-dependent effects in semiconductor lasers were studied at various levels of complexity.[18–21] A transparent rate-equations (RE) approach to QW-based lasers has allowed to elucidate main consequences of the spin-polarized injection.[22] An important finding of this QW model is that the injection threshold J_T, characterizing spin-unpolarized lasers, splits into two thresholds, $J_{T1}<J_{T2}$, when the injected carriers are spin polarized. When injection reaches J_{T1} (majority threshold), the laser starts to emit photons with one helicity (circular polarization), the other helicity joining at J_{T2}, at which minority-spin electrons reach the threshold density. Both experiments[2,3,9] and theory[20,22] have demonstrated an important advantage of the spin lasers over the unpolarized ones: $J_{T1}<J_T$, assuming that all other parameters are identical. The threshold reduction,

$$r = 1 - J_{T1}/J_T, \tag{1}$$

would be largest for fully spin-polarized electrons with infinite spin-relaxation time, reaching as much as $r=5/9$.[22] According to the model, for any injection in the J_{T1} to J_{T2} interval, the laser acts as a "spin filter," i.e., it emits circularly polarized light, even if the spin polarization of injected carriers is small. The relative width of this interval

$$d = (J_{T2} - J_{T1})/J_T \tag{2}$$

increases with the injected spin polarization. The "filtering" effect is another merit of spin lasers, as it offers new opportunities for their dynamic operation. Modulation of injected spin polarization was shown to modulate the intensity of laser emission, even at a constant total injection and to increase the modulation bandwidth.[23]

So far, theoretical description of spin lasers has been essentially limited to QW-based models. To find distinguishing features of QD spin lasers, in this work we formulate a model, which allows for analytical results and offers a direct comparison with the previous results for the QW spin lasers.[2,3,9,22,23] Here, we focus on the parameters motivated by the experiments on (In,Ga)As-based QD spin lasers.[12,21,24] It is instructive, however, to consider other possible materials for spin QW and QD lasers since a variety of active regions has been used for their conventional (spin-unpolarized) counterparts. This choice can be guided by long spin-relaxation time for electrons, which enhances the desirable spin-laser characteristics.[25] Longer spin-relaxation times can result, for example, from a reduction in spin-orbit coupling, one of the main sources of spin relaxation.[1,17] This can be achieved by choosing materials with light elements or by using different growth orientation in QWs.[10] Long spin-relaxation times have been reported in CdSe/ZnSe (an example of a II-VI structure) self-organized QDs.[26] Detailed predictive studies of the spin-relaxation mechanisms in QDs (Refs. 17 and 27) will serve an important role in future efforts in designing QD spin lasers. It would also be interesting to consider active regions with magnetic doping, where the spin degeneracy of the lasing transition may be lifted. II-VI materials doped with Mn are a promising direction since QD lasers based on II-VI structures have already been considered.[28] The problem of the Mn internal transition, which reduces the intensity of band-to-band transitions can be addressed by using ZnSe/(Zn,Mn)Te epitaxial QDs,[29] characterized by a relatively low fundamental transition energy.

A very interesting emerging field is lasers based on colloidal semiconductor QDs [typically II-VI, such as CdS, CdSe, ZnSe, and ZnTe (Refs. 30 and 31)]. These nanostructures are easily synthesized, offer a large tunability of transition energies and a long spin-coherence time.[32] Some colloidal QD structures suffer, however, from the very fast (<100 ps) nonradiative Auger recombination that hinders population inversion and is therefore detrimental for optical gain. This effect can be avoided by using the so-called type-II band alignment in which spatial separation of electrons from holes significantly suppresses the Auger recombination.[33] Just like their self-assembled counterparts, colloidal QDs can be doped magnetically.[34]

II. RATE-EQUATIONS MODEL

The cavity of the QD spin laser is in resonance with interband transitions between QD-confined levels.[12] Since the levels are derived from valence and conduction bands, a general description requires keeping track of both electron and hole populations, as previously shown both for bipolar spintronic devices,[35] and for QD spin-unpolarized lasers.[36] The QDs capture electrons and holes from energy levels of a two-dimensional QW-like wetting layer (WL), which acts as a reservoir of carriers.[37,38] Figure 1 depicts the level structure and the various processes represented by our REs, from carrier injection to photon emission.

We describe the carriers by eight spin-resolved REs coupled to two REs for two circular polarizations of stimulated emission,

$$df_{w\alpha\pm}/dt = I_{\alpha\pm} - C_{\alpha\pm} + \frac{2}{\kappa_\alpha}E_{\alpha\pm} - R_{w\pm} \mp F_{w\alpha}, \tag{3}$$

$$df_{q\alpha\pm}/dt = \frac{\kappa_\alpha}{2}C_{\alpha\pm} - E_{\alpha\pm} - R_{q\pm} - G_{\pm} \mp F_{q\alpha}, \tag{4}$$

$$df_{S\mp}/dt = \Gamma G_{\pm} - f_{S\mp}/\tau_{ph}, \tag{5}$$

cf. Fig. 1. The index w stands for WL and q for QDs while $\alpha = n, p$ for electrons and holes, respectively. Equations (3) and (4) describe carrier occupancies, $0 \le f \le 1$, in WL and QDs, related to the corresponding numbers of particles $n_{w\alpha}$ and $n_{q\alpha}$,

$$f_{w\alpha\pm} = n_{w\alpha\pm}/(N_{w\alpha}/2), \tag{6}$$

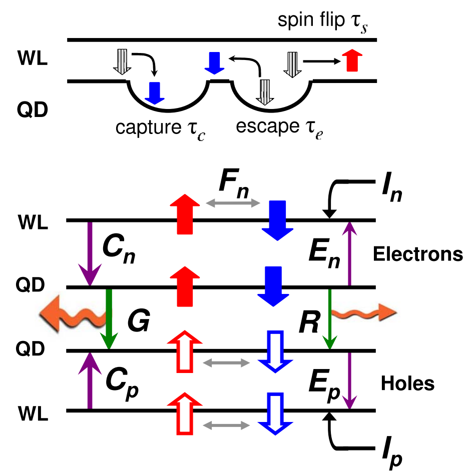

FIG. 1. (Color online) Processes in our model of a spin laser described by Eqs. (3)–(14). QD: quantum dot, WL: wetting layer. Upper panel: thick arrows denote electron spin direction in processes labeled by their corresponding times. Lower panel: thick vertical arrows show the carrier spin (filled for electrons, empty for holes). Curved arrows show carrier injection I. Thin arrows depict capture C, escape E, spin relaxation F, stimulated (G), and spontaneous (R) recombination (thickness indicates relative rates). The subscripts n and p represent the electron and hole contributions, respectively. Wavy arrows depict photon emission.

$$f_{q\alpha\pm} = n_{q\alpha\pm}/N_q. \tag{7}$$

Here $N_{w\alpha}$ is the number of states in WL and N_q is the number of QDs. The ratio $\kappa_\alpha = N_{w\alpha}/N_q$, used in Eqs. (3) and (4), is an important parameter of the QD laser.[39] For simplicity, we assume that each QD hosts one double-degenerate level per species α. This can be realized only for electron levels in small enough QDs (Ref. 40), but we do not expect our results to be qualitatively changed upon inclusion of QD excited states. As long as the lasing transitions involve only QD-confined levels, the limited density of QD states and the limited capture rate will affect the spin-laser characteristics in the way discussed below. The ground state of holes is assumed to be formed predominantly from heavy-hole wave functions. The electron (hole) level is degenerate with respect to spin $\pm 1/2$ (angular momentum $\pm 3/2$) projection.[1]

Equation (5) is for photon occupancies, f_S, of helicities $\mp$, defined as

$$f_{S\pm} = S^{\pm}/N_q, \tag{8}$$

where $S^{\pm}$ is the number of cavity photons of the given helicity. Our sign convention for indices denoting the spin projections and helicities follows Ref. 22. In Eq. (5), Γ is the optical confinement factor and τ_{ph} is the photon cavity lifetime. The terms

$$I_{\alpha\pm} = J_{\alpha\pm}(1 - f_{w\alpha\pm}), \tag{9}$$

$$C_{\alpha\pm} = f_{w\alpha\pm}(1 - f_{q\alpha\pm})/\tau_{c\alpha}, \tag{10}$$

$$E_{\alpha\pm} = f_{q\alpha\pm}(1 - f_{w\alpha\pm})/\tau_{e\alpha} \tag{11}$$

represent carrier injection, carrier capture from the WL to QDs, and the inverse process of escape, respectively. $J_{\alpha\pm} = (1 \pm P_{J\alpha})J_\alpha$ is the number of carriers of α species injected into the laser per WL state of the given spin and unit time with $J_\alpha = (J_{\alpha+} + J_{\alpha-})/2$. The injection spin polarization is $P_{J\alpha} = (J_{\alpha+} - J_{\alpha-})/(J_{\alpha+} + J_{\alpha-})$. The parameters $\tau_{c\alpha}$ and $\tau_{e\alpha}$ are the capture and escape times.

To correctly describe consequences of the small density of QD states, as well as saturation of the WL states at high injection, it is important to include in Eqs. (9)–(11) the Pauli-blocking factors, $(1-f)$, of the WL and QD states.[39] These terms, omitted in some previous work on QD-based spin lasers,[21] impede carrier transfer to states close to saturation. We find that they are particularly important in description of the limited QD occupancies, as shown below.

Defining $\gamma = w, q$, we write the spontaneous radiative recombination in Eqs. (3) and (4) as

$$R_{\gamma\pm} = b_\gamma f_{\gamma n\pm} f_{\gamma p\pm}, \tag{12}$$

where b_γ gives the recombination rate. The spin-relaxation terms

$$F_{\gamma\alpha} = (f_{\gamma\alpha+} - f_{\gamma\alpha-})/\tau_{s\alpha\gamma} \tag{13}$$

equilibrate spin subpopulations with relaxation times $\tau_{s\alpha\gamma}$. A realistic model of a steady-state or dynamic operation of spin lasers, should reflect the different behaviors of electron and hole spins.[22,23] Due to the strong spin-orbit coupling in the valence band, the spin polarization of holes is lost relatively quickly, i.e., $\tau_{sp\gamma} \ll \tau_{sn\gamma}$, both in QWs (i.e., also in the WL) and QDs.[1,9,41] Therefore we assume that the holes, unlike electrons, are spin unpolarized, i.e., $P_{Jp}=0$ and $f_{\gamma p\pm} = f_{\gamma p}$, which implies $I_{p+} = I_{p-}$ in Eq. (9). Additionally, the electron-spin relaxation in QWs is faster than in QDs, thus we take $\tau_{snq} \to \infty$. This a very good approximation at low temperatures,[42] and it remains reasonable at room temperature, where τ_{snq} reaches 1 ns.[43]

The gain term in Eqs. (4) and (5),

$$G_\pm = g(f_{qn\pm} + f_{qp\pm} - 1) f_{S\mp}, \tag{14}$$

describes coupling of the carriers and light, which gives rise to stimulated emission. The sign ordering in subscripts is consistent with the optical selection rules for interband transitions.[1] The constant g is independent of photon occupancies $f_{S\pm}$, i.e., it does not contain the gain compression terms.[44,45] In spite of that, our QD model naturally predicts light-output saturation due to the limited capture capacity of QDs, as discussed below. We note that, owing to the above-mentioned spin asymmetry between electrons and holes, the assumption $f_{qn\pm} = f_{qp\pm}$ is not justified for $P_{Jn} \neq 0$. Thus, an attempt to express, e.g., G_+ [Eq. (14)] using only f_{qn+} (and f_{S-}), would lead to incorrect threshold values, even for the QW spin-laser model.

III. RESULTS

We focus on the steady-state regime, in which the total charge in the spin laser is constant. This imposes a relation between $J_p = J_{p+} = J_{p-}$ and $J_{n\pm}$. One of the REs for carriers then becomes linearly dependent on the others and we replace it with the condition of overall charge neutrality. In the spirit of the simple RE approach, we neglect carrier-carrier Coulomb interactions, which may become important at high injection.[46]

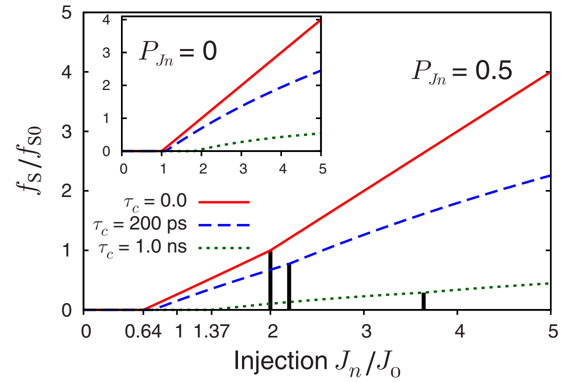

FIG. 2. (Color online) Main panel: dependence of QD spin-laser emission on electron injection, shown for different capture times τ_c. Total photon occupancy, $f_S = (f_{S+} + f_{S-})/2$, is normalized to f_{S0} [Eq. (16)], while the electron injection to J_0 [Eq. (17)]. The parameters are $\tau_{ph} = 1$ ps, $b_q\tau_{ph} = 0.01$, $g\tau_{ph} = 2$, $\kappa = 100$, $\tau_{snw,e} \to \infty$, and $P_{Jn} = 0.5$. Vertical lines denote minority thresholds J_{T2}. The smallest ($\tau_c = 0$) majority threshold J_{T1} for $P_{Jn} = 0.5$, marked at 0.64, gives the threshold reduction $r = 0.36$ [Eq. (1)]. Inset: results for spin-unpolarized lasers, $P_{Jn} = 0$, with the other parameters' values same as in the main panel.

We have obtained all formulas presented below by solving the REs analytically. To give simple expressions that offer insight into the behavior of the spin laser, we assume $\Gamma = 1$, $\kappa_\alpha = \kappa$, $R_{w\pm} = 0$, $\tau_{c\alpha} = \tau_c$, and $\tau_{e\alpha} = \tau_e$.[47] We have checked that the spontaneous-emission coupling to the lasing mode has a negligible effect on our results.[22,23] Thus we set the coupling factor $\beta = 0$. This allows for an unambiguous determination of the laser thresholds.

To develop a preliminary understanding of the QD model of a spin laser, we relate it to the simpler QW model, discussed in Sec. I. In the limit of $\tau_c \to 0$ and $\tau_e \to \infty$, $f_{w\alpha\pm}$ vanish, as can be inferred from Fig. 1. In this case, WL plays no role in the above QD model, which becomes "QW-like," i.e., similar (but not identical) to the QW model of Sec. I. We emphasize that it is not our goal here to compare the absolute thresholds of a QW- and a QD-based laser. Such a comparison requires distinct parameters for these two structures and shows the potential for achieving lower thresholds in the latter.[15,48,49] Here, we use the same range of parameters for the QD model and for its QW-like limit (except for $\tau_{c,e}$). Thus, the QW-like model leads to *lower* thresholds since it describes effectively a QD-based structure in the limit of instant capture. Nevertheless, this approach enables us to elucidate important qualitative differences between the QW- and QD-based spin lasers.

First, we consider consequences of the finite capture rate, $\tau_c > 0$, for a spin-unpolarized laser, $P_{Jn} = 0$, illustrated in the inset of Fig. 2. Let J_T be the threshold for a given τ_c. For any $J_n > J_T$, the QD occupancies are independent of τ_c and fulfill $f_{qn\pm} = f_{qp} = f_0$, where f_0 is the occupancy pinned at the threshold value,[39]

$$f_0 = 1/2 + 1/(2g\tau_{\rm ph}). \tag{15}$$

We normalize the light-injection characteristics using quantities in the limit of instant capture, $\tau_c=0$. The total photon occupancy, $f_S=(f_{S+}+f_{S-})/2$, is expressed in terms of

$$f_{S0} = f_S(\tau_c=0,\ P_{Jn}=0,\ J_n=2J_T) = b_q\tau_{\rm ph}f_0^2 \tag{16}$$

while the injection J_n is normalized to

$$J_0 = J_T(\tau_c=0) = 2b_qf_0^2/\kappa. \tag{17}$$

Unlike the pinned occupancies, J_T increases with τ_c (Fig. 2, inset) as

$$J_T = \left[1 + \frac{2f_0}{\kappa(1-f_0)}\frac{\tau_c}{\tau_e}\right]\frac{J_LJ_0}{J_L-J_0}, \tag{18}$$

where $J_L=(1-f_0)/\tau_c$ is the maximum capture rate C, Eq. (10), realized for $f_{wn}=f_{wp}=1$. The factor (J_L-J_0) in Eq. (18) imposes an upper limit on τ_c above which lasing is impossible ($J_T\to\infty$). The limiting condition, $J_L\geq J_0$, means that J_L must overcome the recombination losses, b_q, determining J_0. When $\tau_c\to 0$, the threshold J_T reduces to J_0 from Eq. (17).

In a model of a QW laser with no gain compression, the laser light intensity depends linearly on injection (we neglect the small deviations from linearity that appear around the thresholds when the coupling factor $\beta>0$). A linear dependence is also found for the QD model with $\tau_c=0$. In contrast, the QD model with $\tau_c>0$ reveals a sublinear dependence (Fig. 2, inset), even though we do not introduce any gain-compression terms.[50] At higher injection the emission saturates, as discussed below for the spin-polarized injection scenario.

Next, we turn to the spin-polarized injection, i.e., $P_{Jn}\neq 0$. Similarly to the QW model from Sec. I, our QD model predicts two lasing thresholds,[51] $J_{T1}<J_{T2}$, as shown in Fig. 2, main panel. We find that, in general, the increase in J_{T1} and J_{T2} with τ_c is quantitatively similar to the increase in J_T. A particularly simple example is the minority threshold in the limit of $\tau_{snw}\to\infty$,

$$J_{T2}/J_T = 1/(1-|P_{Jn}|) \tag{19}$$

valid for any τ_c, τ_e, b_q, g, $\tau_{\rm ph}$, κ, and identical to the relation found for the QW-based laser.

Such simple, universal relations are typical for the QW model, but not for the QD one with $\tau_c>0$. Even with the simplifying assumptions: $b_q\tau_c\ll 1$, $g\tau_{\rm ph}=2$, large κ (i.e., $f_{wn\pm},f_{wp}\ll 1$), and $\tau_{snw,e}\to\infty$, we obtain a more complicated ratio for the majority threshold,

$$\frac{J_{T1}}{J_T} = \frac{4}{(2+|P_{Jn}|)^2}\left[1+\frac{18|P_{Jn}^3|b_q\tau_c}{1+6|P_{Jn}|+3P_{Jn}^2-10|P_{Jn}^3|}\right], \tag{20}$$

showing that the threshold reduction r, Eq. (1), depends on τ_c. Equation (20) reduces to the simple QW-model result for $\tau_c=0$ [Eq. (4) of Ref. 22 in the limit $w\to 0$, i.e., infinite spin-relaxation time].

Figure 3 shows the evolution of J_{T1} and J_{T2} as a function of the capture time. We use a range of τ_c, which reflects the scope of values found in previous works.[36,52] We start from an initial set of parameters: $\tau_{\rm ph}=1$ ps,[2] $b_q\tau_{\rm ph}=0.01$,[53] $g\tau_{\rm ph}=2$,[54] $\kappa=100$,[55] $P_{Jn}=0.5$, $\tau_{snw,e}\to\infty$, and then we vary some of the values to determine the relevant trends. The limit $\tau_{snw}\to\infty$ enables us to obtain analytical formulas [such as Eq. (20)], we also present numerical results for $\tau_{snw}=100$ and 200 ps, i.e., the order of magnitude found in experiments.[56]

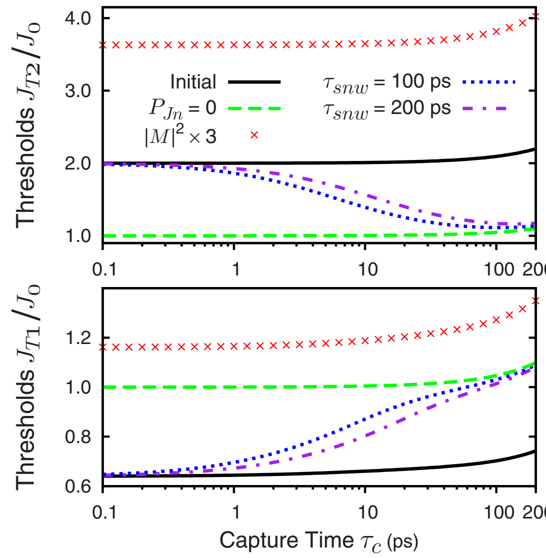

FIG. 3. (Color online) Majority and minority (lower and upper panel) thresholds of a QD spin-polarized laser. The solid line shows our result for the initial parameters given in Fig. 2. The crosses, dashed-dotted, and dotted lines are the thresholds when one of the parameters is changed (see legend). A threefold increase in the squared modulus of the optical matrix element, $|M|^2$, results in a threefold increase in both b_q and g. Dashed line in both panels is J_T for $P_{Jn}=0$. The normalizing current $J_0=J_T(\tau_c=0)$, Eq. (17), has been calculated for fixed parameters (the initial parameters of Fig. 2, except $P_{Jn}=0$).

Both J_{T1} and J_{T2} increase with τ_c since the capture rate into the QDs, Eq. (10), decreases. Comparing J_{T1} to J_T (solid and dashed line, lower panel), we note only a small decrease in the threshold reduction defined in Eq. (1); $r(\tau_c=200$ ps$)=0.32$ versus $r(\tau_c=0)=0.36$. Using these values, we calculate the "spin-filtering" interval, Eq. (2), from Eqs. (1) and (19) for $P_{Jn}=0.5$. It decreases monotonically from the maximum $d=1.36$ for $\tau_c=0$ to $d=1.32$ for $\tau_c=200$ ps, only a small shrinking of the filtering region.

In the limit of $\tau_{snw}\to\infty$, we find that J_{T1}, J_{T2}, and J_T rise uniformly with decreasing capture rate for a wide range of parameters, e.g., see the solid, dashed, and crosses line in Fig. 3. For decreasing τ_{snw}/τ_c, however, both J_{T1} and J_{T2} approach J_T (dotted and dashed-dotted line), so the values of r and d decrease. If the time that the electrons spend in the WL is not much shorter than τ_{snw}, their spin polarization will be largely erased before capture by the QDs. The typical times, $\tau_c\sim 1-10$ ps, make this scenario unlikely.

The influence of escape time τ_e on the thresholds is modest. Keeping the ratio $\delta=\tau_c/\tau_e$ fixed, and increasing τ_c, we find similar shifts of J_{T1} and J_{T2} to slightly higher values. By changing δ from 0 to 1.25 (zero- to high-temperature limit[39]), the spin-filtering region decreases from $d(\tau_c=10$ ps$)=1.34$ to $d(\tau_c=10$ ps$)=1.32$, with similar changes in the $0.1 < \tau_c < 200$ ps interval (the other parameters retaining the initial values).

It is interesting to consider the influence of the optical matrix element of the lasing transition, M. Increasing $|M|^2$ results in a proportional increase of both g and b_q,[45] representing gain and radiative losses, respectively. The increase in the losses prevails so that all the thresholds rise. The value of $J_T(\tau_c=0)$ is an example: in Eq. (17) f_0 decreases with increasing g (increasing $|M|^2$) to the minimum 1/2, but b_q grows indefinitely. Figure 3 shows the corresponding change in J_{T1} and J_{T2} on the example of a threefold increase in $|M|^2$. The matrix element modifies $J_{T1,2}$ to a different extent than J_T. With growing $|M|^2$, the threshold J_T rises faster, which results in a higher r and d. For example, setting $\tau_c=10$ ps and using the initial parameters, except for $\tau_{snw}=100$ ps (appropriate for room temperature[56]), we find $r=0.13$ and $d=0.52$. These values increase to $r=0.22$, $d=0.75$ for $g\tau_{\mathrm{ph}}=8$ and $b_q\tau_{\mathrm{ph}}=0.04$ (a fourfold increase in $|M|^2$). We find a similar improvement of r with increasing photon lifetime τ_{ph}. The above value of $r=0.13$ rises to 0.22, when τ_{ph} changes from 1 to 4 ps. Thus, the detrimental effect of spin relaxation in WL can be mitigated by modifying laser parameters not related to spin.

Apart from increasing the thresholds, the limited supply of carriers to the lasing transition causes output saturation. This can be understood by looking at the regime of high injection. High J_n drives the WL occupancies close to saturation, $f_{wn\pm}=f_{wp}=1$, because the finite τ_c limits carrier relaxation to QDs. In this regime, the capture rates approach their maxima, J_L [see Eq. (18)], so that the injection into QDs no longer grows with J_n. The asymptotic value of photon occupancy

$$f_{\mathrm{S}}^{\max} \equiv [f_{\mathrm{S}+}(J_n \to \infty) + f_{\mathrm{S}-}(J_n \to \infty)]/2 < \infty \qquad (21)$$

is independent of P_{Jn}.[51] We obtain $f_{\mathrm{S}}^{\max} \sim 1/\tau_c$ for $J_L \gg J_0$. Interestingly, $f_{\mathrm{S}+}(J_n\to\infty)=f_{\mathrm{S}-}(J_n\to\infty)$, so that the circular polarization of laser light, $P_{\mathrm{S}} \equiv (f_{\mathrm{S}+}-f_{\mathrm{S}-})/(f_{\mathrm{S}+}+f_{\mathrm{S}-})$, is zero for high injection, in contrast to the QW model, where $P_{\mathrm{S}} \to -P_{Jn}$.[22] This can be explained as follows. In a QW-laser model with no gain compression term, levels participating in the laser action are assumed to be replenished instantaneously (a characteristic relaxation time is ~ 1 ps, Ref. 44). The capture process to the discrete, widely spaced QD levels is slower,[46] and must be treated explicitly in a realistic QD model. As noted above, this leads to $f_{wn\pm} \lesssim 1$ at sufficiently high J_n, so the electrons captured into the QDs are spin unpolarized and consequently $P_{\mathrm{S}} \to 0$.

Finally, we note that the limited capture rate is not the only difference between QW- and QD-based lasers. Since $f_{q\alpha\pm} \le 1$, Eq. (15) imposes a lower limit on the gain required for lasing: $g\tau_{\mathrm{ph}} \ge 1$ for any P_{Jn}, τ_c, and the other parameters. The QW model of Sec. I predicts no such limit. A more restrictive condition must be satisfied to maintain the full threshold reduction: $g\tau_{\mathrm{ph}} \ge 1+|P_{Jn}|$ in the $\tau_c \to 0$ limit. Decreasing $g\tau_{\mathrm{ph}}$ below $1+|P_{Jn}|$ results in a decrease in r, which vanishes completely, when $g\tau_{\mathrm{ph}} \to 1$. These effects are a direct consequence of the limited density of states at the lasing transition, a limitation that can be neglected in QW-based lasers operating at low powers. We also note that the upper bound $f_{\gamma\alpha} \le 1$ must be enforced by including Pauli-blocking terms, otherwise the REs lead to incorrect results, also for $P_{Jn}=0$. For example, if the $1-f_q$ term in Eq. (10) is omitted (f_q is any of the equal QD occupancies), the REs allow for the unphysical $f_q > 1$, so that J_T is always reached, even when $g\tau_{\mathrm{ph}} < 1$. For $g\tau_{\mathrm{ph}} \gtrsim 1$ and $\tau_c \sim 200$ ps, the omission of $1-f_q$ leads to relative errors of J_T as high as 30%.

IV. CONCLUSIONS

In this work we have developed a transparent rate-equation approach, which has allowed for analytical results. Using this formalism, we have elucidated various trends in operation of QD spin lasers, comparing them to their relatively well-known QW-based counterparts. In particular, we have studied the consequences of finite capture rate by QD-confined levels, which participate in the lasing transition. To fully preserve the threshold reduction and the spin-filtering effects resulting from spin injection, the capture time has to be much shorter than the spin-relaxation time in the wetting layer. Nevertheless, we have found that, when the spin relaxation lowers the electron-spin polarization appreciably, the threshold reduction and the spin-filtering window can be partially restored by modifying some spin-independent laser parameters. Another consequence of the finite capture rate is saturation of stimulated emission as a function of injection. Furthermore, QD- and QW-based lasers have qualitatively different densities of the initial and final states of lasing transitions. The threshold reduction in QD lasers may be hindered by the small density of QD states, if the gain g or the photon cavity lifetime is too small.

To take full advantage of the potential of electrical spin injection in QD spin lasers, it is important to further improve their magnetic contacts (injectors). The current maximum temperature of 200 K for electrically injected spin lasers using MnAs injector,[12] will likely be soon improved, since the same spin injector material was recently demonstrated to operate at room temperature.[57] Fe Schottky contacts have also been used to inject spins in (In,Ga)As QDs at room temperature.[58] For the surface-emitting spin lasers, magnetic injectors with out-of-plane remanent magnetization would be desirable.[59] In such a geometry, the spin-laser operation is possible without the need to apply an external magnetic field, since the optical selection rules lead to circularly polarized light.[1] Encouraging results have been reported recently for spin light-emitting diodes utilizing MgO tunnel contacts,[59] which provide a very efficient room-temperature spin injection.[60]

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